Student registration number: 189141
Exam name: Microeconomics

Part I: Multiple Choice Questions:

Question	Answer
1	b
2	b
3	d
4	b
5	e
6	d
7	d
8	c
9	d
10	a
11	b
12	d
13	a
14	c
15	d
16	c

Part II:
I answered questions: 1 and 2

1.

Case 1: Firm 1 don't invest:

$$profit\ function\ of\ firm\ 1:$$
$$\pi 1 = \left(600 - \frac{Q}{2} - 300\right) * q1 - 10000$$
$$= \left(300 - \frac{q1 + q2}{2}\right) * q1 - 10000$$
$$Maximizing\ profit:$$

$$\frac{d\pi 1}{dq1} = 0 \rightarrow 300 - q1 - \left(\frac{q2}{2}\right) = 0$$
$$Solving\ for\ q1:$$
$$q1 = 300 - \frac{q2}{2}$$
$$By\ symmetry\ q1 = q2:$$

$$q1 = 200 - \frac{q1}{300}$$
$$q1 = q2 = 200$$
$$\pi 1 = \pi 2 = \left(600 - \frac{(200 + 200)}{2} - 300\right) * 200 - 10000 = \mathbf{10000}$$

Case 2: Firm 1 invest:

$profit\ function\ of\ firm\ 1$:

$$\pi 1 = \left(600 - \frac{Q}{2} - 200\right) * q1 - 10000$$
$$= \left(400 - \frac{q1 + q2}{2}\right) * q1 - 10000$$

$Maximizing\ profit$:

$$\frac{d\pi 1}{dq1} = 0 \rightarrow 400 - q1 - \left(\frac{q2}{2}\right) = 0$$
$$\rightarrow \boldsymbol{q1 = 400 - \frac{q2}{2}}$$

$profit\ function\ of\ firm\ 2$:

$$\pi 2 = \left(600 - \frac{Q}{2} - 300\right) * q2 - 10000$$
$$= \left(300 - \frac{q1 + q2}{2}\right) * q2 - 10000$$

$Maximizing\ profit$:

$$\frac{d\pi 2}{dq2} = 0 \rightarrow 300 - q2 - \left(\frac{q1}{2}\right) = 0$$
$$\rightarrow \boldsymbol{q2 = 300 - \frac{q1}{2}}$$

Solving the system:

$$q2 = 300 - \frac{q1}{2}$$
$$q1 = 400 - \frac{q2}{2}$$
$$\rightarrow \boldsymbol{q2 = 133.333}, \quad \boldsymbol{q1 = 333.335}$$

Using this to find profits:

$$\pi 1 = \left(600 - \frac{(133{,}333 + 333{,}335)}{2} - 200\right) * 333{,}335 - 10000$$
$$= \mathbf{45555.6}$$
$$\pi 2 = \left(600 - \frac{(133{,}333 + 333{,}335)}{2} - 300\right) * 133{,}333 - 10000$$
$$= \mathbf{-1111.42}$$

Firm 1 gaining's after investment: $45555.6 - 20000 = 25555.6$

So Firm 1 will invest since 25555.6 > 10000

b.
Now firm 1s investment will give Firm b an advantage aswell:

$$profit\ function\ of\ firm\ 1:$$
$$\pi 1 = \left(600 - \frac{Q}{2} - 200\right) * q1 - 10000$$
$$= \left(400 - \frac{q1 + q2}{2}\right) * q1 - 10000$$
$$Maximizing\ profit:$$

$$\frac{d\pi 1}{dq1} = 0 \rightarrow 400 - q1 - \left(\frac{q2}{2}\right) = 0$$
$$Solving\ for\ q1:$$
$$q1 = 400 - \frac{q2}{2}$$
$$By\ symmetry\ q1 = q2:$$
$$q1 = 400 - \frac{q1}{2}$$
$$\boldsymbol{q1 = q2 = 266.667}$$
$$\boldsymbol{Using\ this\ to\ find\ profits:}$$
$$\pi 1 = \pi 2 = \left(600 - \frac{(266.667 + 266.667)}{2} - 200\right) * 266.667 - 10000$$
$$= 25555.5$$

However Firm 1 actual profit after investment will be:

$$25555.5 - 20000 = \mathbf{5555.5}$$

Company 1 will not invest in the improvement since 10000>5555.5

c.
The value of the investment is larger for firm 1 when it only benefits Firm 1 themselves. In question b the investment has a positive externality on Firm

B; it decreases the marginal cost for both Firms, leading to an increase in the duopoly's profits. So that the value of the investment is no longer fully captured by the firm undertaking the investment explains the differences in value of the investment for Firm 1.

2.

To produce in Denmark:

Producing q>0 requires all inputs (interior solution). Since K is fixed, at the optimal input choice : $\frac{MPl}{w} = \frac{MPm}{m}$

$$MPl = 0.5 * L^{-0.5} * M^{0.5} * 10$$
$$MPm = 0.5 * L^{0.5} * M^{-0.5} * 10$$

Optimizing inputs require:

$$\frac{0.5 * L^{-0.5} * M^{0.5} * 10}{400} = \frac{0.5 * L^{0.5} * M^{-0.5} * 10}{100}$$
$$Using\ this\ we\ get\ that: M = 4L$$
$$Thus\ Q = L^{0.5} * (4L)^{0.5} * 10$$
$$Q = 20 * L \rightarrow L = \frac{Q}{20}, M = 4 * \left(\frac{Q}{20}\right)$$

$$C = 400L + 100M + 10 * 100$$
$$Substituting\ into\ cost\ function:$$
$$C = 400 * \left(\frac{Q}{20}\right) + 100 * \left(4 * \left(\frac{Q}{20}\right)\right) + 1000$$
$$\boldsymbol{C = 800 * \left(\frac{Q}{20}\right) + 1000 = 40Q + 1000}$$

Economies of scale?

AC(q)=$\frac{40Q+1000}{q}$ →

Ac decreases with q, thus the cost function exhibits economie of scale.

Profits?

C(q)= $800 * \left(\frac{Q}{20}\right) + 1000 \rightarrow MC = 40$

$$profit = (60 - 0.1Q - 40)Q - 1000$$
$$Maximizing:$$
$$\frac{dprofit}{dq} = 0 \rightarrow 20 - 0.2Q = 0, Q = 100, P = 50$$

$$profit = (60 - 0.1 * 100 - 40)100 - 1000 = \mathbf{0}$$

Poland :

$$MPl = 0.5 * L^{-0.5} * M^{0.5} * 10$$
$$MPm = 0.5 * L^{0.5} * M^{-0.5} * 10$$

Optimizing inputs require:

$$\frac{0.5 * L^{-0.5} * M^{0.5} * 10}{200} = \frac{0.5 * L^{0.5} * M^{-0.5} * 10}{50}$$

Thus: $\mathbf{M = 4L}$

$substituting\ into\ production\ function$: $Q = L^{0.5} * (4L)^{0.5} * 10$

$$Q = 20 * L \rightarrow L = \frac{Q}{20}, M = 4 * \left(\frac{Q}{20}\right)$$

$$C = 200L + 50M + 10 * 100$$

$Substituting\ into\ cost\ function$:

$$C = 200 * \left(\frac{Q}{20}\right) + 50 * \left(4 * \left(\frac{Q}{20}\right)\right) + 1000$$

$$\boldsymbol{C = 400 * \left(\frac{Q}{20}\right) + 1000 = 20Q + 100}$$

Economies of scale?

AC= $\frac{20Q+100}{Q} = 20 + \frac{100}{Q}$

Thus, the cost function has increasing economies of scale since AC decreases with Q.

Profits?

C(q)= $20Q + 1000 \rightarrow MC = 20$

$$profit = (60 - 0.1Q - 20)Q - 1000$$
$$Maximizing:$$
$$\frac{dprofit}{dq} = 0 \rightarrow 40 - 0.2Q = 0, Q = 200, P = 40$$
$$profit = (60 - 0.1 * 100 - 20)100 - 1000 = \mathbf{2000}$$

So they will choose to produce in Poland since profit of 2000 is larger than 0. Since K is fixed they will use combination of inputs: (10,40,10)
M= 4L
K=10

$$200 = L^{0.5} * (4L)^{0.5} * 10$$
$$\boldsymbol{L = 10}$$
$$\boldsymbol{M = 4 * 10 = 40}$$
$$\boldsymbol{K = 10}$$

b.

L and K is fixed:

$$f(L, M, K) = 10^{0.5} * M^{0.5} * 10^{0.5} = 10M^{0.5}$$
$$Q = 10M^{0.5} \rightarrow M = \frac{Q^2}{100}$$
$$Substitute\ this\ into\ cost\ function:$$
$$C(q) = 10 * 200 + 50 * \frac{Q^2}{100} + 10 * 100 = \mathbf{3000} + \frac{\boldsymbol{Q^2}}{\mathbf{2}}$$

Economies of scale?

This function exhibits increasing economies of scale at low outputs, but at higher outputs the AC increase with Q, leading to diseconomy of scale.

c.

Profit in Poland when L and K is fixed:

$C(q) = 3000 + \frac{Q^2}{2}$

$$MC = Q$$
$$P = Q$$
$$profit: P(Q) * Q - C(Q)$$
$$(60 - 0.1Q)Q - \left(3000 + \frac{Q^2}{2}\right)$$
$$Maximizing\ by\ setting\ derivative = 0$$
$$\rightarrow Q = 50$$
$$(60 - 0.1 * 50)50 - \left(3000 + \frac{50^2}{2}\right) = \boldsymbol{-1500}$$

So then the firm will choose to produce in Denmark since 0 profit is better than negative profit.

Demand change:

Denmark profit:

$$profit = (60 - 0.001Q - 40)Q - 1000$$
$$\text{At } 1000 = Q$$
$$profit = (60 - 0.001 * 1000 - 40)1000 - 1000 = 18000$$

Poland profit (if M and L are freely adjusted):

$$profit = (60 - 0.001Q - 20)Q - 1000$$

$$\text{At Q} = 1000$$

$$profit = (60 - 0.001 * 1000 - 20)1000 - 1000 = 38000$$

Poland profit if only M is freely adjusted:

$$(60 - 0.001Q)Q - \left(3000 + \frac{Q^2}{2}\right)$$

$$\text{At Q} = 1000$$

$$(60 - 0.001 * 1000)1000 - \left(3000 + \frac{1000^2}{2}\right) = -444000$$

So the firm will choose to produce in Poland if L and M is variable, if only M is variable they will choose to produce in Denmark.
When all inputs are not variable, they cannot choose to produce with the optimal inputs making the costs increase. When only M is variable for production in Poland the costfunction exhibits diseconomy of scale making it hard to scale production since the costs increase with output. Therefore, it is important to choose a country where the inputs are variable so that they can lower cost and increase profit.