Question 1

a) What is the annual operating cash flow in each year of the project?

The annual operating cash flow per year is inserted below, and this is afterwards elaborated on a yearly basis:

	Year 1	Year 2	Year 3	Year 4
Operating cash flow in $	15.28	17.9	19.48	17.75

In year 1, the operating cash flow is $15.28. The revenue is $40 and COGS is 60% of that which is $24 (40*0.6=24). The depreciation per year is:

$$\frac{Purchase\ price - salvage\ value}{Useful\ Life} = \frac{120}{10} = \$12\ per\ year$$

The pre-tax profit is thus $4, since 40-36 = $4. We then calculate the tax by multiplying the pre-tax profit with the tax rate of 18%. Thus, the taxes paid in year 1 is 4*0.18 = $0.72.

The after-tax profit is thus $3.28, and we then add back depreciation of $12 to get to the operating cash flow in year 1 of $15.28. See table below

Annual operating cash flow year 1		Calculations
Revenue	40.00	=40
Less opportunity costs	0.00	
- Expenses (Sum of below)	-36.00	
COGS	-24.00	=40*-0.6
Fixed costs	0.00	
*Depreciation**	-12.00	=-(120-0)/10
Pretax profit	4.00	
- Taxes (marginal taxe rate * pretax profit)	-0.72	=4*-0.18
After tax profit	3.28	
Depreciation	12.00	
Annual Operating Cash flow	**15.28**	=After tax profit + Depreciation

For year 2, the operating cash flow is $17.9. Revenue has increased by 20% so the new revenue is 40*1.2=$48, and the COGS has resultingly increased since it is 60% of revenue. The new COGS is 48*0.6 = $28.8. The depreciation is the same as last year (12 dollars). The taxes paid are 7.2*0.18 = $1.3. This gives an after-tax profit of $5.9 and then we add back depreciation of 12 dollars and end with the operating cash flow of $17.9. See table below.

Annual operating cash flow year 2		Calculations
Revenue	48.00	=48
Less opportunity costs	0.00	
- Expenses (Sum of below)	-40.80	
COGS	-28.80	=48*-0.6
Fixed costs	0.00	
*Depreciation**	-12.00	=-(120-0)/10
Pretax profit	7.20	
- Taxes (marginal taxe rate * pretax profit)	-1.30	=7.20*-0.18
After tax profit	5.90	
Depreciation	12.00	
Annual Operating Cash flow	**17.90**	=After tax profit + Depreciation

For year 3, the operating cash flow is $19.48. We follow the same methodology as before, and in this year revenue increased by 10% from last year to €52.8, which then gives a COGS of 52.8*0.6 = $31.68. Resultingly, the taxes paid are $9.12*0.18 = $1.64 and the after-tax profit is thus $7.48. And then we add back depreciation of $12 to get to $19.48. See table below.

Annual operating cash flow year 3		Calculations
Revenue	52.80	=52.8
Less opportunity costs	0.00	
- Expenses (Sum of below)	-43.68	
COGS	-31.68	=52.8*-0.6
Fixed costs	0.00	
*Depreciation**	-12.00	=-(120-0)/10
Pretax profit	9.12	
- Taxes (marginal taxe rate * pretax profit)	-1.64	=9.12*-0.18
After tax profit	7.48	
Depreciation	12.00	
Annual Operating Cash flow	**19.48**	=After tax profit + Depreciation

For year 4, the operating cash flow is $17.75. We realize that revenue has decreased to $47.52 (52.8*0.9 = $47.52) and this resultingly affects the COGS and makes it -$28.51 (47.52*0.6=$28.51), and the pre-tax profit is thus 9.12$. This affects the taxes paid which are then 9.12*0.18 = 1.64. See table below.

Annual operating cash flow year 4		Calculations
Revenue	47.52	=47.52
Less opportunity costs	0.00	
- Expenses (Sum of below)	-40.51	
COGS	-28.51	=47.52*-0.6
Fixed costs	0.00	
*Depreciation**	-12.00	=-(120-0)/10
Pretax profit	7.01	
- Taxes (marginal taxe rate * pretax profit)	-1.26	=7.01*-0.18
After tax profit	5.75	
Depreciation	12.00	
Annual Operating Cash flow	**17.75**	=After tax profit + Depreciation

It is also mentioned in the text that *"If the project is undertaken, the project will for accounting reasons only need to share the fixed fees (which are already paid for) related to an existing IT system. These costs will be $20".*

<u>We do not take this cost into account, since they are already paid for and it would be for accounting reasons (not related to cash flows).</u>

b) **What is the total cash flow from the project in years 0, 1, 2, 3 and 4?**

The total cash flow from the project in years 0, 1, 2, 3 and 4 is summarized in the table, and then the calculations are done per year afterwards.

	Year 0	Year 1	Year 2	Year 3	Year 4
Total yearly cash flow in $	-132	15.28	15.504	18.038	71.15

<u>In year 0</u>, the cash outflow is the sum of the initial investment (-120) and the working capital in year 0 which is -12:

$$Year\ 0\ cashflow = -120 - 12 = -\$132$$

<u>In year 1</u> (where the annual operating cash flow is $15.28), we also have to account for changes in working capital.

So the total cash flow from the project is:

$$Year\ 1\ cashflow = Operating\ cash\ flow - investment\ in\ working\ capital - investment\ in\ fixed\ assets$$

In year 0, the working capital was set to $12, and in year 1 it should be maintained at 30% of sales. So it should be 40 * 0.3 = $12. Thus, in year 1, the working capital does not increase. And there are no additional investments in fixed assets for any of the years.

The year 1 cashflow is thus:

$$Year\ 1\ cashflow = 15.28 - 0 = \$\mathbf{15.28}$$

In year 2, we do the same:

$$Year\ 2\ cashflow = Operating\ cash\ flow - investment\ in\ working\ capital - investment\ in\ fixed\ assets$$

The operating cash flow is $17.9.

The working capital should be maintained at 30% of sales, which were $48, and for year 1 the working capital was 12. The increase in working capital is thus: $(48 * 0.3) - 12 = \$\mathbf{2.4}$

There are no new investment in fixed assets. Thus:

$$Year\ 2\ cashflow = 17.9 - 2.4 = \$\mathbf{15.5}$$

In year 3, we again follow the same formula:

$$Year\ 3\ cashflow = Operating\ cash\ flow - investment\ in\ working\ capital - investment\ in\ fixed\ assets$$

The operating cash flow is $19.48.

The increase in working capital is: $(52.8 * 0.3) - 14.4 = \$\mathbf{1.44.}$ Since the working capital should be maintained at 30% of sales, which were $52.8, and for year 2 the working capital was 12+2.4=14.4.

There are no new investment in fixed assets. Thus:

$$Year\ 3\ cashflow = 19.48 - 1.44 = \$\mathbf{18.038}$$

Calculating the year 4 cash flow:

$$Year\ 4\ cashflow = Operating\ cash\ flow - investment\ in\ working\ capital - investment\ in\ fixed\ assets$$

The operating cash flow is $17.75.

In year 4, the working capital is fully freed up. So, the cash flow from working capital is positive and equal to 12+2.4+1.44=$15.84.

In year 4, the fixed asset is also sold for 30 dollars. At this time, the book value of the asset was:

$$Book\ value = purchase\ price - (yearly\ depreciation * years)$$

$$120 - 12 * 4 = \$72$$

Thus, the loss on the sale of the asset is:

$$Loss\ on\ asset = Book\ value - sales\ price$$

$$Loss\ on\ asset = 72 - 30 = 42$$

And the tax shield from this loss is:

$$42 * tax\ rate = 42 * 0.18 = 7.56$$

Thus, the cash flow from selling the asset is: Sales price + tax benefits =

$Cash\ flow\ from\ selling\ asset = 30\ +\ 7.56\ =\ \$37.56.$

And the year 4 cash-flow is thus:

$$Year\ 4\ cashflow = 17.75 + 15.84 + 37.56 = \$\mathbf{71.15}$$

c) **What is the NPV and the dynamic pay-back period of the project?**

Now that we have calculated the yearly cash flows, we can calculate the NPV by discounting them back to the present value, using the discount rate of 6% (r=0.06). We use the formula below:

$$NPV = C_0 - \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \cdots + \frac{CF_t}{(1+r)^t}$$

$$NPV = -132 - \frac{15.28}{(1+0.06)^1} + \frac{15.504}{(1+0.06)^2} + \frac{18.038}{(1+0.06)^3} + \frac{71.15}{(1+0.06)^4}$$

$$NPV = -32.28$$

Thus, the NPV is negative, and the project should not be done.

<u>The dynamic payback period</u> cannot be calculated, since the NPV of the whole project is negative, and we can thus not calculate how long it takes for the project's cash flow (when discounted) to equal NPV=0.

Question 2:

a)

Firstly, we calculate the PV of the annuity, where t=40, C= 10,000 kr. And r=0.01=10%.

$$PV = C \cdot \left(\frac{1}{r} - \frac{1}{r(1+r)^t}\right)$$

We insert the numbers:

$$PV = 10000\left(\frac{1}{0.1} - \frac{1}{0.1(1+0.1)^{40}}\right)$$

$$PV = 97{,}790.507$$

Since we have to calculate the retirement savings account 40 years from now, we then find the Future value of the present value, using the formula:

$$FV = PV * (1+r)^t$$

We insert the present value and t and r which are the same as listed above:

$$FV = 97{,}790.507 * (1+0.1)^{40}$$

$$FV = 4{,}425{,}925.56$$

<u>Thus, the retirement savings is expected to be DKK 4,425,925.56 in 40 years from now</u>

b)

We calculate the fair expected return of fund A with a Beta of 1.5, and use the CAPM model for this:

$$r_{equity} = r_f + \beta(r_m - r_f)$$

We also know the return of the market is 8% and the risk-free rate is 1%:

$$r_{equity} = 0.01 + 1.5(0.08 - 0.01)$$

$$r_{equity} = 0.115 = \mathbf{11.5}\%$$

Thus, the fair expected return of Fund A is 11.5% given its level of risk (beta).

Then, we calculate the fair expected return of fund B with a beta of 0.75, and again use the CAPM model for this:

$$r_{equity} = r_f + \beta(r_m - r_f)$$

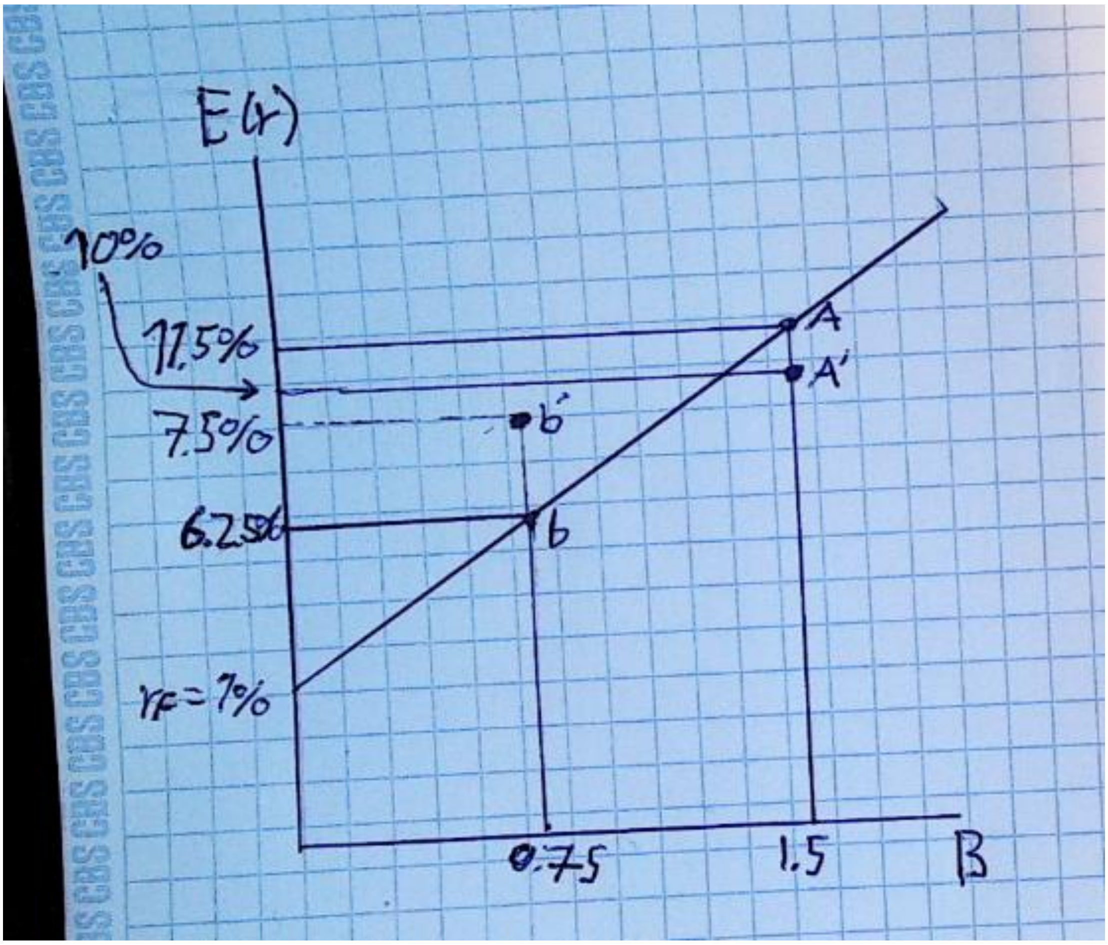
We also know the return of the market is 8% and the risk-free rate is 1%:

$$r_{equity} = 0.01 + 0.75(0.08 - 0.01)$$

$$r_{equity} = 0.0625 = \mathbf{6.25}\%$$

Thus, the fair expected return of Fund B is 6.25% given its level of risk (beta).

We now plot these in the security-market line:



Here b is the fair expected return for fund B (6.25%) and A is the fair expected return for fund A (11.5%). Both of them are on the Security-market line. The actual expected returns of the funds (denoted A' and b') can then be seen that they are not on the line. The actual expected return for fund b is higher (b'=7.5%) and for fund A it is lower and equal to 10% (A').

So, Fund B is underpriced, and Fund A is overpriced. Given risk-return perspectives**, you should invest in fund B** since it has a higher return than what could be expected given the level of risk (and the reverse is true for fund A, that has a lower return than what could be expected given the level of risk). For good measure it is also worth noting that the $Y - intercept = r_f = 1\%$.

c)

Since fund A has a Beta of 1.5 and fund B has a beta of 0.75, you cannot invest in fund b and invest in a risk-free asset and thus get to a Beta of 1.5 (since risk-free asset implies that the Beta is equal to 0).

The question is thus understood as if, it asks us to invest in Fund A, and a risk-free asset to get to a Beta equal to 0.75 (which is the Beta for fund B).

We calculate the Beta of such a portfolio using the formula:

$$B_p = (\%A) * \beta_a + (\%risk\ free) * \beta_{risk-free}$$

We know that the end Beta will have to be 0.75 (Beta for fund B). Additionally, we know that $\beta_a = 1.5$ and $\beta_{risk-free} = 0$ since $\beta_{risk-free}$ $is\ not\ affected\ by\ market\ movements$.

We use the following formula to solve for the share of the portfolio that should be Fund A stocks:

$$Percent\ of\ fund\ A = \frac{End\ Beta - Beta\ of\ riskfree\ asset}{Fund\ A\ beta - \ Beta\ of\ riskfree\ asset}$$

$$Percent\ of\ fund\ A = \frac{0.75 - 0}{1.5 - 0} = 0.5 = 50\%$$

Thus, 50% of the portfolio should be in Fund A. **And since there are only 2 assets in the portfolio to reach the Beta of 0.75, then we know that 50% of the portfolio should be in the risk-free asset.**

Thus:

$$B_p = 0.5 * 0.15 + 0.5 * 0 = \mathbf{0.75}$$

And this beta of the portfolio is equal to Fund B's beta.

The expected return of such a portfolio. Where fund A's return is 10%, and the return on the risk-free asset is 1%, is calculated by using the formula where p_r is the return on the portfolio and $r_A = the\ actual\ annual\ average\ return\ of\ fund\ A$:

$$P_r = (\%\ in\ riskfree\ asset) * r_f + (\%\ in\ fund\ A) * r_A$$

We calculate the expected return of the Portfolio:

$$P_r = 0.5 * 0.01 + 0.5 * 0.1 = 0.055 = \mathbf{5.5}\%$$

Thus, this is the average annual expected returns.

Interpreting the results, we see that this return the same Beta as fund B (Beta=0.75), but only gives an expected return of 5.5%, which is below the security-market line (such that it would be called overpriced and provide lower returns than expected given the level of risk). Fund B on the other hand, is expected to yield 7.5%.