Exam Number: 20-12-2024
Exam name: Microeconomics

Part I: Multiple Choice Questions:

Question	Answer
1	d
2	e
3	b
4	a
5	d
6	c
7	e
8	b
9	c
10	a
11	d
12	d
13	b
14	d
15	b
16	d

Part II:

I answered questions: 1 and 2.

1.

Two firms, A and B, serve the market. They have the production functions

$$f^A(L, M, K) = L^{0.4}M^{0.4}K^{0.2}$$

$$f^B(L, M, K) = \min\{L, M, K\}$$

Labor, materials and capital cost

$$w = 20, m = 20, r = 10$$

For each unit.

a)

To find the cost function for A, I first find the optimal combination:

$$MP_L = 0.4 * \frac{f}{L}$$

$$MP_M = 0.4 * \frac{f}{M}$$

$$MP_K = 0.2 * \frac{f}{K}$$

The marginal products divided by cost should be equal, so we get the same amount out of every input:

$$\frac{MP_L}{w} = \frac{MP_M}{m} = \frac{MP_K}{r}$$

$$\frac{0.4 * \frac{f}{L}}{20} = \frac{0.4 * \frac{f}{M}}{20} = \frac{0.2 * \frac{f}{K}}{10}$$

$$\frac{0.4}{20L} = \frac{0.4}{20M} = \frac{0.2}{10K}$$

$$L = M = K$$

Meaning the optimal combination is all inputs being equal.

$$q = f^A(L, L, L) = L^{0.4}L^{0.4}L^{0.2} = L$$

And the cost function becomes:

$$C_A(q) = 20(q) + 20(q) + 10(q) = 50q$$

This cost function has constant economies of scale, as it is linear.

Since the inputs for firm B are perfect compliments, they should be used in the same proportion. To produce one unit, firm B needs to spend 20 on L, 20 on M and 10 on K. Thus, the cost function becomes:

$$C_B(q) = 50q$$

This cost function has constant economies of scale, as it is linear.

Since L = M = K, the optimal combination of inputs for firm A to produce 100 units is:

$$f^A(L, L, L) = 100$$

$$L^{0.4}L^{0.4}L^{0.2} = 100$$

$$L = 100$$

The optimal input combination is for all inputs to be 100, L = 100, M = 100 and K = 100.

b)

To find a short-run cost function for firm A, I set K = 32. From the previous analysis, L = M.

$$q = 32^{0.2}L^{0.4}L^{0.4} = 2L^{0.8}$$

This means that:

$$L = \sqrt[0.8]{\frac{q}{2}} = \left(\frac{q}{2}\right)^{\frac{1}{0.8}}$$

The short-run cost function then becomes:

$$C_A(q) = 20\left(\frac{q}{2}\right)^{\frac{1}{0.8}} + 20\left(\frac{q}{2}\right)^{\frac{1}{0.8}} + 10(32) = 320 + 40\left(\frac{q}{2}\right)^{\frac{1}{0.8}}$$

The AVC is increasing in q, meaning decreasing returns to scale.

The short-run production function of firm A can be written as:

$$f^A(L, M, 32) = 2L^{0.4}M^{0.4}$$

If we multiply it by 2, we get

$$f^A(2L, 2M, 2 * 32) = 2^{0.2} * 32^{0.2} * 2^{0.4}L^{0.4} * 2^{0.4}M^{0.4} = 2 * 2L^{0.4}M^{0.4}$$
$$= 4L^{0.4}M^{0.4}$$

The production function has constant returns to scale.

The short-run cost function of firm B is the same as the long-run cost function, since it can still vary all inputs.

c)

Market demand is given by:

$$Q = 100 - 0.5P$$

meaning that:

$$P = 200 - 2Q$$

The profit functions are:

For firms of type A:

$$\pi_A(q) = (200 - 2Q)q - 50q$$

For firms of type B:

$$\pi_B(q) = (200 - 2Q)q - 50q$$

I use the formula for Cournot Nash Equilibriums:

$$q_1 = \frac{a - 2MC_1 + MC_2}{3b},\ q_2 = \frac{a - 2MC_2 + MC_1}{3b}$$

Since there are 4 firms with the same profit function competing, Q = 4q.

$$q_1 = \frac{200 - 2 * 50 + 50 + 50 + 50}{3 * 8} = 10.42$$

By the same logic, all quantities should be the same:

$$q_1 = q_2 = q_3 = q_4$$

Price is then given by:

$$P = 200 - 2 * (4 * 10.42) = 116.67$$

And the profit for each individual firm is:

$$\pi = (116.67 - 50) * 10.42 = 694.45$$

The Cournot Nash Equilibrium quantity and profit for each firm is 10.42 and 694.45.

2.

The inverse demands for ordinary (o) and fancy (f) gløgg are:

$$P_0 = 100 - 2Q_o - Q_f, P_f = 200 - 2Q_f - Q_o$$

Where Q_o and Q_f are the total number of units of ordinary and fancy gløgg, respectively, and prices are in kroner. Firm 1 and 2 serve the market with the same production costs of:

$$MC_o = 10, MC_f = 40$$

a)

If both firms sell fancy gløgg, the Cournot Nash Equilibrium is:

$$q_1 = \frac{a - 2MC_1 + MC_2}{3b}, \ q_2 = \frac{a - 2MC_2 + MC_1}{3b}$$

$$q_1 = \frac{200 - 2 * 40 + 40}{3 * 2} = 26.67$$

$$q_2 = \frac{200 - 2 * 40 + 40}{3 * 2} = 26.67$$

The price is then:

$$P = 200 - 2 * (2 * 26.67) = 93.33$$

And the profit for each firm is:

$$\pi = (93.33 - 40) * 26.67 = 1422.24$$

b)

Now, Firm 1 serves ordinary gløgg while Firm 2 serves fancy gløgg.

To find the Nash Equilibrium, I set up the profit functions, differentiate and set equal to 0 to find the equations for Q_o and Q_f. I then solve them as two equations with two unknows.

Setting up the profit functions:

$$\pi_1(Q_o, Q_f) = (100 - 2Q_o - Q_f)Q_o - 10Q_o$$

$$\pi_2(Q_o, Q_f) = (200 - 2Q_f - Q_o)Q_f - 40Q_f$$

Differentiating the first and setting equal to 0:

$$\frac{\delta\pi_1}{\delta Q_o} = 100 - 4Q_o - Q_f - 10 = 90 - 4Q_o - Q_f$$

$$\frac{\delta\pi_1}{\delta Q_o} = 0 \Leftrightarrow 90 - 4Q_o - Q_f = 0 \Leftrightarrow Q_o = 22.5 - \frac{Q_f}{4}$$

Differentiating the second and setting equal to 0:

$$\frac{\delta\pi_2}{\delta Q_f} = 200 - 4Q_f - Q_o - 40 = 160 - 4Q_f - Q_o$$

$$\frac{\delta\pi_2}{\delta Q_f} = 0 \Leftrightarrow 160 - 4Q_f - Q_o = 0 \Leftrightarrow Q_f = 40 - \frac{Q_o}{4}$$

The two equations with two unknowns are then:

$$Q_o = 22.5 - \frac{Q_f}{4}$$

$$Q_f = 40 - \frac{Q_o}{4}$$

Solving for Q_o:

$$Q_o = 22.5 - \frac{1}{4}(40 - \frac{1}{4}Q_o) \Leftrightarrow Q_o = 12.5 + \frac{1}{16}Q_o \Leftrightarrow 0.9375Q_o = 12.5$$

$$\Leftrightarrow Q_o = 13.33$$

With that, I can also find Q_f:

$$Q_f = 40 - \frac{1}{4} * 13.33 = 36.67$$

The corresponding profits are then:

$$\pi_1(13.33, 36.67) = (100 - 2 * 13.33 - 36.67) * 13.33 - 10 * 13.33$$
$$= 355.51$$

$$\pi_2(13.33, 36.67) = (200 - 2 * 36.67 - 13.33) * 36.67 - 40 * 36.67$$
$$= 2689.01$$

c)

Whatever Firm 1 chooses, Firm 2 will choose the other one. If they choose the same kind of gløgg and compete by setting price simultaneously, they will have to price at MC, which is the same for both firms. The resulting profit will then be 0. We have the equilibrium from the two firms choosing different kinds from b). We can then make a profit table of the possible options:

(Firm 1, Firm 2)	Ordinary	Fancy
Ordinary	(0, 0)	(355.51, 2689.01)
Fancy	(2689.01, 355.51)	(0,0)

Firm 1 knows that if it chooses fancy gløgg, the only way for Firm 2 to make a profit will be to choose ordinary gløgg. Since (fancy, ordinary) has the highest profit for Firm 1, they will choose to produce fancy gløgg. The equilibrium is then Firm 1 and Firm 2 choosing (fancy, ordinary) gløgg and making a profit of (2689.01, 355.51).