Multiple Choice

1	D
2	E
3	В
4	A
5	С
6	D
7	E
8	В
9	С
10	A
11	E
12	С
13	A
14	D
15	В
16	С

Longer Questions

Question 2

a. If both firms decide to sell fancy gløgg, the demand curve will be as follows

$$P_f = 200 - 2Q_f - Q_o(0) = 200 - 2Q_f$$

As we are dealing with a linear demand, a now homogenous market and oligopolistic behavior, we can find the Nash Equilibrium using our Cournot formula for identical firms given $q_1 = \frac{a-m}{n+1(h)}$

The quantities will therefore be as follows

$$q_1 = \frac{200 - 40}{(2+1)(2)} = 26.67$$

$$q_2 = \frac{200 - 40}{(2+1)(2)} = 26.67$$

The corresponding profits will therefore be

$$\pi_1 = 26.67(200 - 2(26.67)) - 26.67(40) = 2844.62$$

$$\pi_2 = 26.67(200 - 2(26.67)) - 26.67(40) = 2844.62$$

b. If Firm 1 decides to produce ordinary gløgg and Firm 2 decides to produce fancy gløgg, we are now dealing with a differentiated oligopoly, and we will have to derive our best responses from the profit functions.

The profit function are as follows

$$\pi_{1(Q_o)} = Q_o (100 - 2Q_o - Q_f) - 10Q_o$$

$$\pi_{2(Q_f)} = Q_f (200 - 2Q_f - Q_o) - 40Q_o$$

Take the derivate of each using the product rule

$$\frac{\partial \pi_1}{\partial Q_o} = 100 - 2Q_o - Q_f - 2Q_o - 10 = 90 - 4Q_o - Q_f$$

$$\frac{\partial \pi_2}{\partial Q_g} = 200 - 2Q_f - Q_o - 2Q_f - 40 = 160 - 4Q_f - Q_o$$

Set to zero and solve for the respective Qs

$$4Q_o = 90 - Q_f \to Q_o = 22.5 - 0.25Q_f$$

$$4Q_f = 160 - Q_o \rightarrow Q_o = 40 - 0.25Q_o$$

Insert one best response into the other

$$Q_o = 22.5 - 0.25(40 - 0.25Q_o) = 22.5 - 10 + 0.0625Q_o$$

 $0.9375Q_o = 12.5 \rightarrow Q_o = 13.33$
 $Q_g = 40 - 0.25(13.33) = 36.67$

The profits in this case would then be

$$\pi_1 = 13.33(100 - 2(13.33) - 36.67) - 10(13.33) = 356.51$$

$$\pi_2 = 36.67(200 - 2(36.67) - 13.33) - 30(36.67) = 3055.71$$

c. To see the possible profits of the different firms I have created a table with the corresponding profits of the different actions of the two firms.

I will quickly calculate the profit in a simultaneous game where they both choose to produce ordinary gløgg.

$$q_1 = \frac{100 - 10}{(2+1)(2)} = 15$$

$$q_2 = \frac{100 - 10}{(2+1)(2)} = 15$$

The corresponding profits will therefore be

$$\pi_1 \& \pi_2 = 15(100 - 2(15)) - 15(10) = 900$$

	Firm 1		
		Fancy	Ordinary
		2844.62	356.51
Firm 2	Fancy	2844.62	3055.71
Fi		3055.71	900
	Ordinary	356.51	900

The possible actions are

- Firm 1 chooses ordinary, therefore Firm 2 chooses fancy giving firm 1 a profit of 356.51
- Firm 1 chooses fancy, therefore Firm 2 chooses fancy giving firm 1 a profit of 2844.62.

The Subgame Perfect Nash Equilibrium is therefore where Firm 1 chooses to produce fancy gløgg and Firm 2 chooses to produce fancy gløgg, yielding a profit of (2844.62 and 2844.62.

Question 3

a. To find the aggregate demand of the first good, we use the following formula as we are dealing with a cobb-douglas formula

$$x = \frac{a}{a+b} \frac{B}{p_x}$$
For Albert $x = \frac{0.4}{0.4+0.6} \frac{1000}{p_x} = \frac{400}{p_x}$
For Bertha $x = \frac{0.6}{0.4+0.6} \frac{1000}{p_x} = \frac{600}{p_x}$

As this is the aggregate demand, we need to multiply each demand with 1000.

$$Q = 1000 \frac{400}{p_r} + 1000 \frac{600}{p_r} = \frac{1000^2}{P}$$

b. First, we need to reverse the supply function

$$P = 0.16Q \rightarrow Q = 6.25P$$

We then set it equal to the aggregate market supply to find the equilibrium price

$$6.25P = \frac{1000^2}{P}$$

$$P = 400$$

We then insert our price into our demand function to find that

$$Q(400) = \frac{1000^2}{400}$$

$$Q = 2500$$

c. To find the social optimal level, we must first include the marginal damage to the supply function.

$$0.16Q + 0.09Q = 0.25Q$$

We then set this equal to the demand again, but we must remember to reverse the demand function first.

$$Q = \frac{1000^2}{P} \to P = \frac{1000^2}{Q}$$
$$0.25Q = \frac{1000^2}{Q}$$
$$Q = 2000$$

The new social output is lower, as we have included the negative externality. As the externality have a negative impact on society, it will shift the supply curve inwards, creating a new equilibrium with a lower input and higher price.

d. At the optimal level, the total output would have to be 2000.

The price paid could therefore be found inserting this into the supply function therefore have to be

$$2000 = \frac{1000^2}{P}$$

$$P = 500$$

To find the tax needed to ensure this, we would have to set the new price equal to the supply curve with an added tax.

$$500 = \frac{2000}{16} + t$$
$$t = 375$$

First, we find utility level in the competitive market for Bertha

Given the budget constraints and $p_x = 400$ the consumption of each item will be

$$x = \frac{0.6}{0.4 + 0.6} \frac{1000}{400} = 1.5$$

$$y = \frac{0.4}{0.4 + 0.6} \frac{1000}{100} = 4$$

$$U(x, y) = 1.5^{0.6}4^{0.4} = 2.22$$

We know find budget given our new price of good 1 at 500.

$$2.22 = \left(0.6 \frac{B}{500}\right)^{0.6} \left(0.4 \frac{B}{100}\right)^{0.4}$$

$$B = 1142.93$$

Thus, the compensating variation to consumers like Bertha needed to attain the same utility level will be 142.93 *euros*.