Multiple choice questions:

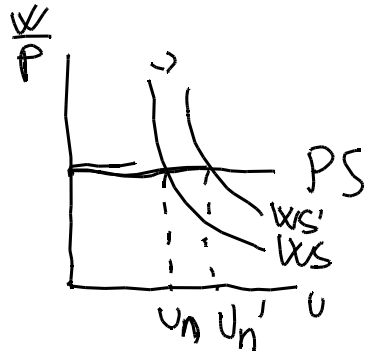
1	a
2	c
3	b
4	e
5	d
6	a

Exercise 1: Investment shock and inflation

a. **Assume that targeted inflation is equal to 2%. Use the fitted line equation given above to calculate the natural rate of unemployment over this period.**
Since the natural rate (middle run equilibrium) then we now that inflation $\pi_t = \pi^e{}_t = \overline{\pi_t}$. Therefore, in the formula, we can plug the targeted inflation of 2% into the current inflation (on the left side), and solve for the natural rate of unemployment.
$2 = 3 - 0.2 * u_t \leftrightarrow -1 = -0.2 * u_t \leftrightarrow u_t = 5$. The natural rate of unemployment is 5%.

b. **Explain why the natural rate of unemployment moves over time and varies across countries. Choose one of the determinants of the natural rate of unemployment seen in class and explain how a change in this determinant affects the natural rate of unemployment. Show graphically using the wage-setting/price-setting graph.**
The natural rate of unemployment is a rate that is undefined, because it does not have an exact value, but it is usually the value of unemployment that is often seen in a country over a long time, where the economy is not changing that much. The current unemployment goes up and down during recessions and when the economy is expanding/expansionary (during good, positive economy), respectively. If the economy has had high unemployment for many years in a row, it would makes sense that the natural rate of unemployment also increases and vice versa.

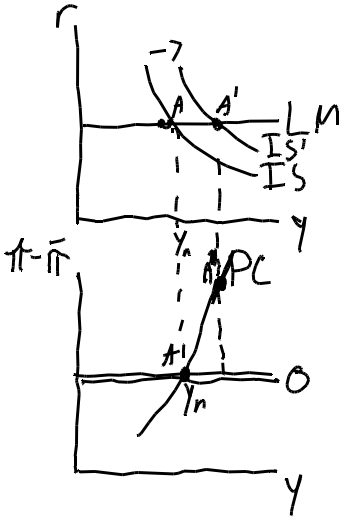
In the EU there is a lot of benefits for the labor force (unemployment benefits, minimum wages, work protection, etc.) which increases the z variable (all catch variable), and therefore this goes up, it leads to a shift on the WS (wage setting) curve to the right, leading to a higher level of natural rate of unemployment. In the US, they don't have that many unemployment benefits, etc. and therefore they are at a lower natural rate of unemployment, because they don't have the same "increase" in the z variable. The increase in z variable for EU, can be seen below, as the WS curve shift to the right, leading to a higher natural rate of unemployment:



c. **Describe in detail the effect of this change in bo on output, unemployment, the nominal wage, and investments, before any potential intervention of the central bank. Explain why inflation is affected by this change in investor confidence. Describe graphically using the IS-LM-PC graph.**

When b0 increases, investment will increase, and we know that output will go up. When output goes up, unemployment decreases. When unemployment decreases, wages go up (workers have more bargaining power – can be seen in the WS equation), and this leads to an increase in prices (can be seen in the PS equation). When prices increase, inflation also increases.

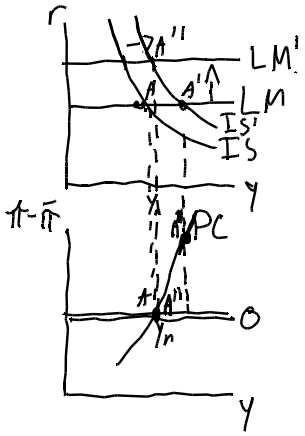
On the IS-LM-PC curve, the IS curve will shift to the right, due to the increase in investment, leading to a higher output, and we move along the PC, where we reach a higher inflation level. This can be seen below, where we go from A to A' on both IS-LM curve and in PC curve:



d. **Explain why the central bank would decide to intervene. What is the appropriate policy response? How will the main variables evolve? Explain and represent graphically using the IS-LM-PC graph.**

Since inflation goes above the natural rate of inflation, the central bank would like to intervene, to bring it back to normal level. Inflation will lead to increase in prices and therefore the central bank would bring it back. In order for CE (central bank) to bring back

inflation, they would have to increase the interest rate, in order for output to fall again. An increase in the policy rate, would increase the real interest rate, leading to lower investment (as it becomes more expensive to loan money), leading to decrease in output (also lower consumption). As output decreases, unemployment will increase, which leads to lower wages (WS equation), which leads to lower prices (PS equation) and which leads to a decrease of inflation, back to the natural rate of inflation.
This can be seen below, where the LM curve shifts upwards, leading to point A'', where output comes back to the initial level, and the same with inflation:



e. **How do the levels of inflation, consumption, and investment compare to their initial levels, i.e. before the change in investor confidence?**
Inflation is at the same level, consumption is at the same level, since output is the same. Investment is also at the initial level, since the increase in b0 has been canceled out by the increase in real interest rate (which leads to higher borrowing rates for investment), and therefore investment has fallen back to the initial level.

Exercise 2: IS-LM model (short run)

a. Derive the IS and LM equations. Find the equilibrium level of output, consumption and investment.

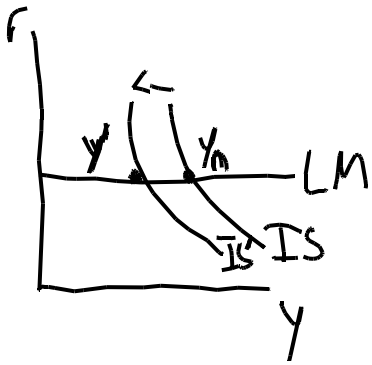
The IS curve:
$Y = Z$, where $Z = C + I + G$, therefore: $Y = C + I + G$. Insert C and I: $Y = c_0 + c_1(Y - T) + b_0 + b_1 Y - b_2(r + x) + G = c_0 + c_1 Y - c_1 T + b_0 + b_1 Y - b_2(r + x) + G$. Isolate Y and c1y and b1y: $Y - c_1 Y - b_1 Y = c_0 - c_1 T + b_0 - b_2(r + x) + G \leftrightarrow Y(1 - c_1 - b_1) = c_0 - c_1 T + b_0 - b_2(r + x) + G$. Isolate Y: $Y(1 - c_1 - b_1) = c_0 - c_1 T + b_0 - b_2(r + x) + G \leftrightarrow Y = \frac{1}{(1 - c_1 - b_1)} * (c_0 - c_1 T + b_0 - b_2(r + x) + G)$. Plugging in numbers; $Y = \frac{1}{(1 - 0.6 - 0.2)} * (400 - 0.6 * 500 + 200 - 2000(r + 0) + 600) = 5 * (900 - 2000(r) = 4500 - 10000(r)$. This is the IS relation.

The LM relation is the value of r, which is given as $r = 0.01$

Therefore, output is equal to: $Y = 4500 - 10000 * (0.01) = 4500 - 100 = 4400$
Consumption is: $C = 400 + 0.6 * (4400 - 500) = 400 + 2340 = 2740$
Investment is: $I = 200 + 0.2 * 4400 - 2000 * (0.01) = 1060$

b. Suppose that consumers become more pessimistic towards future developments in the economy: the consumer confidence index falls. Consumer confidence being proxied by $c0$, we observe that $c0$ decreases from 400 to 300. Calculate the new equilibrium level of output and show the effect of this event in the IS-LM diagram. Explain verbally how a decrease in consumer confidence affects output and the other endogenous variables.

When c0 goes down by 100, and we have a multiplier of 5, it means that Y goes down by 500:
$Y = \frac{1}{(1-0.6-0.2)} * (300 - 0.6 * 500 + 200 - 2000(r + 0) + 600) = 5 * (900 - 2000(r) = 4000 - 10000(0.01) = 3900$. The new output is therefore 3900.
As c0 decreases this means that the IS curve will shift leftwards, leading to a lower output, seen below:



When c0 decreases, it leads to consumption decreases, leading to output going down, and investment will also decrease as it depends on output. Government spending will be the same.

c. Suppose that, after the decrease in consumer confidence, the government wants to bring output back to its initial level. Explain verbally how a fiscal policy would affect output. Calculate the level of taxes T that would bring output back to the original level (with G unchanged). Assume that the Ricardian equivalence does not hold.

The government wants to bring output back to the initial point, by doing an expansionary fiscal policy, by decreasing T, and not changing G. As T lowers, people would consume more, because disposable income goes up, which leads to an increase in Y, and also an increase in I. Government spending is unchanged.

The new level of T is, when the want to get back to output equal 4400:
$4400 = \frac{1}{(1-0.6-0.2)} * (300 - 0.6 * T + 200 - 2000(r + 0) + 600) = 5 * (1100 - 2000(0.01) - 0.6T) = 5400 - 3T$. Therefore: $4400 = 5400 - 3T \leftrightarrow 3T = 1000 \leftrightarrow T \approx 333.33$

d. Now, suppose that the Ricardian equivalence holds (fully). Explain verbally how output would be affected by the fiscal policy discussed in the previous question. Is the government able to bring output back to its initial level by changing taxes? Why or why not? No algebra needed.

Ricardian equivalence means that because there is a reduction in T now, it is going to be paid back in the near future, and therefore people would instead of spending money, they would save it for future tax increases. Therefore, consumption would decrease, which will make output constant even with a tax reduction. Therefore, the output would be stuck at 3900, because consumption goes down. The government would not be able to bring output back, because of the people decreasing consumption.

e. Write down the law of motion of the (public) debt-to-GDP ratio. In this equation, r denotes the real interest rate on government bonds and g denotes the growth rate of GDP. Assume that the debt-to-GDP ratio was initially quite high. Explain verbally how the fiscal policy discussed in the previous questions would affect the debt-to-GDP ratio. Explain verbally how this evolution in the debt-to-GDP ratio could lead to an increase in the perceived risk of default. Explain how a higher risk of default could affect the interest rate, the debt-to GDP ratio, the primary deficit-to-GDP ratio and output.

The law of motion of public debt-to-GDP ratio is: $\frac{B_t}{Y_t} = (1 + (r + x) - g) * \frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$. In the following question, it was seen that T was decreased, but because of Ricardian equivalence, output does not increase, because people save them instead, because they know that the government would increase taxes later on. In the equation, we would see that output is constant, taxes is going down leading to a higher primary deficit (more government expenses than tax income), leading to a higher debt to GDP ratio. The growth of GDP is also 0 since output does not change, which means that the parenthesis will be equal to (1+r). Normally change in output can decrease debt if it increases higher than the interest rate, but here it does not, since there is no growth. Overall, the Ricardian equivalence and the expansionary fiscal policy leads to a higher debt-to-GDP ratio.

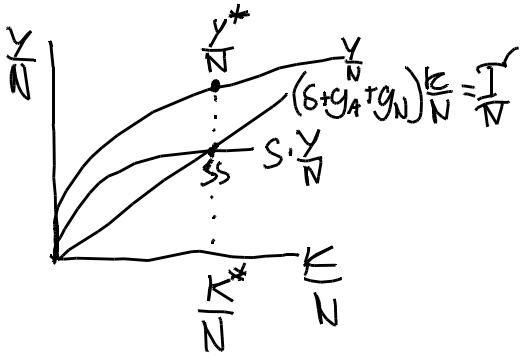
If the debt to GDP is already very high, people might be worried that the government cannot pay back the debt. And since there is no growth in output (GDP), the interest rate and the primary deficit would just feed on the debt to GDP, which makes it higher. Therefore, some would worry about a default situation. When there is a higher risk of defaulting, the x variable in borrowing rate will increase, which makes it more expensive to investment and loan money. It also makes it more expensive for the government to pay back it's debt since the borrowing rate increases, and therefore the debt-to-GDP ratio also increases. As interest rate/borrowing rate increases, output will decline, since investment goes down, output goes down and consumption goes down, which also affect the primary deficit to GDP since Y goes down, leading to a higher primary deficit, leading to a higher debt-to-GDP ratio.

Exercise 3: Solow model

a) Write down the law of motion of capital per worker and use it to derive the equation describing the steady state. In a graph, draw output per worker, investment per worker, and required investment per worker. Explain the shape of these three curves. Show the steady state of this economy graphically. What is the growth rate of output per worker at the steady state? What is the growth rate of output at the steady state? Explain.

The law of motion of capital per worker is: $\frac{K}{N} = \frac{I}{N} - (\delta + g_N + g_A)\frac{K}{N}$, since there is both population growth and technology growth, but they are both constant. To find the equation

that describes the steady state, the change in capital per worker is equal to 0: $\Delta\frac{K}{N} = \frac{I}{N} - (\delta + g_N + g_A)\frac{K}{N} = 0 \leftrightarrow \frac{I}{N} = (\delta + g_N + g_A)\frac{K}{N}$. Investment per worker can be substituted with the savings rate times the output per worker.
The graph would look like this, with output per worker, investment (savings) per worker, and required investment per worker ($\frac{I^r}{N} = (\delta + g_N + g_A)\frac{K}{N}$):

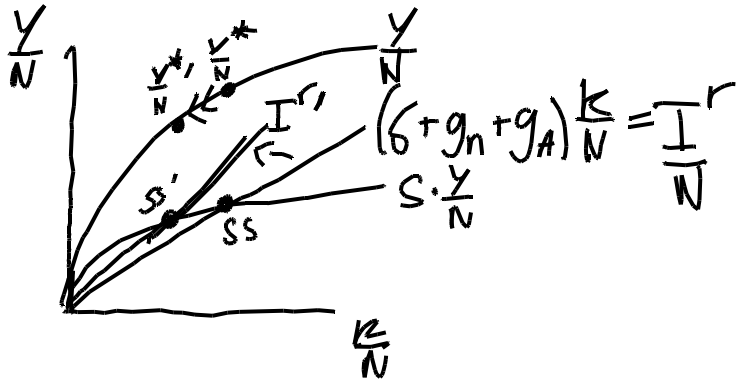


We se that Y/N, output per worker, increases over time, but by less and less (decreasing marginal returns to capital), and therefore it has a concave slope and shape. The savings (investment) per worker is a fraction (as it is multiplied by s) of output per worker, and therefore has the same concave shape and the slope is a fraction of. The required investment per worker is a linear graph with the slope of $(\delta + g_N + g_A)$.

The steady state is where the investment (savings) per worker intersect the required investment per worker, at the point SS on the graph, and the point on output per worker graph is the steady state value of output per worker noted $\frac{Y^*}{N}$.

The growth rate of output per worker at the steady state is g_A, since the growth of population will be canceled out in per worker terms since output is divided by N, and therefore the only growth is g_A.
The growth rate of output at the steady state is $g_A + g_N$, since output here is not divided by N, it also grows at the rate of the population and therefore output grows at the rate of: $g_A + g_N$

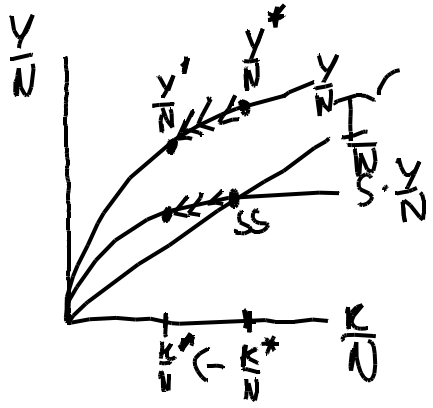
b) Case 1. Suppose that the economy is initially at its steady state. Show in a graph the effects of an increase in the population growth rate. Explain how this event affects the level and the growth rate of output per worker, both when the event occurs and in the long run (i.e. once the economy reaches its steady state).
When there is an increase in population growth rate, the slope will increase in the required investment slope, which makes it pivot to the left leading to a new steady state that is lower than before. This can be seen blow where we move from SS with the steady state of output per worker equal to $\frac{Y^*}{N}$ (also with a steady state value of capital per worker), to a new steady state of SS' and a steady state of output per worker equal to $\frac{Y^{*\prime}}{N}$ (and a new steady state value of capital per worker), which has a lower value as we move along the output per worker graph.



When the event occurs, output per worker will decrease (also capital per worker), with the growth rate faster than g_A, until it reaches the new steady state, where it will only have a growth rate of g_A. The reason it decreases is because the required investment is larger than investment (savings) and therefore there is a negative growth towards the new steady state.

c) Case 2. Suppose that the economy is initially at its steady state. Show in a graph the effects of an earthquake that destroys part of the capital stock. Explain how this event affects the level and the growth rate of output per worker, both when the event occurs and in the long run (i.e. once the economy reaches its steady state)

When an earthquake destroys capital stock, we move along the curve with the output per worker, since Investment required and savings rate does not change. We move to the new level of output per worker and capital per worker, and the economy is below it's steady state. This can be seen below:



When the event occurs, output per worker decreases until we reach the new point of $\frac{Y'}{N}$, but we don't reach a steady state, as we are below the steady state. Therefore, after the earthquake we would see an increase in output per worker that is faster than g_A, until it reaches it's old steady state value, where it will grow at the rate of g_A again. The reason for an increase, when the earthquake is done, is because investments is larger than the required investment, which leads to a faster growth.

d) In light of the Solow model (with no technological growth), is there a tendency for perworker output levels in different countries to approach each other in the long term? Why or why not?

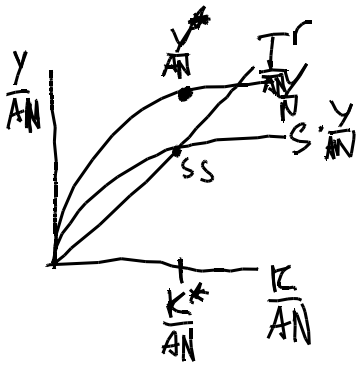
I think that "long term" is a bit undefined, if it means "long run" or "not long run," and therefore I gave a solution for both. But I think you refer to the second, where some countries are not at their steady state, and some are, so convergence.

It depends on what the "long terms" is referred to. If it means "long run" which is equal to steady state, countries would not approach each other since countries have no growth at the steady state (because of no technological growth), and that countries have different steady state, e.g., the US have higher steady states values than China.

But if "long terms" means that we are not at the steady state (so no long run), then countries that are below steady state, will grow faster than countries at the steady state (0 growth at steady state, because of no technological growth), which is called convergence. Countries below steady state will grow at a fast rate since investment is higher than depreciation, and therefore it will come closer and closer to the steady state. For instance, the Western countries have for a long time had low growth rate, which could be a result of being in their steady state (or close to), where China, over many years have experienced a high growth rate, meaning that they are below steady state. However, as they mover closer, the growth rate becomes less and less, and it would take many years for the convergence to happen.

e) Now we assume that there is technological progress: the state of technology is increasing at the rate gA, which is constant. Show the steady state of this economy in the Solow graph, where capital per effective worker is shown on the x-axis. [It is not requested to derive the equation describing the steady state]. What is the growth rate of output per worker at the steady state? What is the growth rate of output at the steady state? Explain.

Since it is now per effect worker it is $\frac{Y}{AN}$. The graph of the Solow model is below, where the point SS is where the savings rate per effective worker and required investment per effective worker intersects (crosses) each other, and we have a steady state value of $\frac{Y^*}{AN}$ and $\frac{K^*}{AN}$:



Output per worker grows at g_A, and output grows at the rate of $g_N + g_A$. Since the question doesn't ask about per effective worker, we know that in output per worker, the growth of population is canceled out with the growth of capital with increasing population, since N and Y grows by g_N, and therefore there is also only g_A left. With output, the population growth is not canceled out since there is no N, and therefore it has the growth of $g_N + g_A$.