Part I: Multiple Choice Questions:

Question	Answer
1	A
2	B
3	C
4	A
5	A
6	C
7	D
8	C
9	A
10	C
11	C
12	B
13	B
14	A
15	B
16	C
17	D
18	B
19	B
20	C

Part II:
I answered questions: 1. Demand Side & 3. Duopolies and Game Theory

1. **Demand Side**

a)
$Marginal\ utility\ of\ wine\ is\ found\ by\ differentiating\ the\ utility\ function\ with\ respect\ to\ W$:

$$U = 2OW$$
$$\boldsymbol{MU_W = 2O}$$

$$If\ the\ amount\ of\ oumph\ doubled:$$
$$U = 2 * 2O * W$$
$$\boldsymbol{MU_W = 4O}$$

$$If\ the\ amount\ of\ wine\ doubled:$$
$$U = 2O * 2W$$
$$\boldsymbol{MU_W = 4O}$$

b)

$$Langrangian:$$
$$Objective: Maximize\ utility\ and\ Constraint: Budget$$
$$I\ have\ her\ utility\ function, and\ need\ to\ find\ her\ budget\ constraint:$$
$$400 = 25O + 25W$$

$$I\ can\ then\ set\ up\ the\ Langrangian\ as\ follows:$$

$$\mathcal{L} = 2OW + \lambda(400 - 25O - 25W)$$

$$Then\ to\ find\ the\ first - order\ conditions\ I\ take\ partial\ derivatives\ of\ \mathcal{L}:$$

$$FOCs:$$

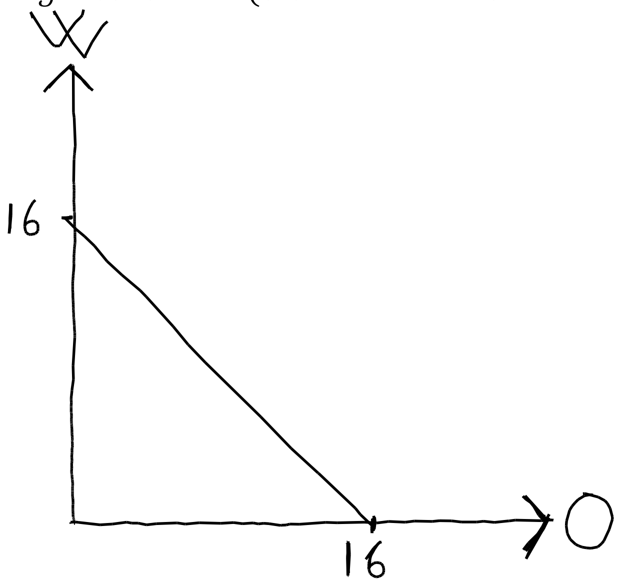
1. $\dfrac{d\mathcal{L}}{dW} = 2O - 25\lambda = 0$
2. $\dfrac{d\mathcal{L}}{dO} = 2W - 25\lambda = 0$
3. $\dfrac{d\mathcal{L}}{d\lambda} = 400 - 25O - 25W = 0$

c)

$$I\ found\ the\ budget\ constraint\ in\ the\ last\ problen:$$
$$I.e: Her\ allowance = P_W * W + P_O * O$$
$$\boldsymbol{Budget\ constraint: 400 = 25O + 25W}$$

$$Her\ MRT\ is\ found\ kie\ this:$$
$$MRT = -\frac{P_O}{P_W} = -\frac{25}{25} = -1$$
MRT=-1

$$Ilustration\ of\ budget\ constraint(O\ on\ the\ x - axis\ and\ W\ on\ the\ y - axis):$$



d)

$$To\ find\ her\ optimal\ consumption\ we\ can\ set\ MRS = MRT$$
$$MRT = -1$$
$$MRS = -\frac{MU_O}{MU_W} = -\frac{2W}{2O} = -\frac{W}{O}$$

$$Set\ MRS = MRT:$$
$$-\frac{W}{O} = -1$$
$$W = O$$

$$I\ can\ now\ plot\ W\ into\ the\ budget\ constraint\ and\ solve\ for\ O:$$
$$400 = 25O + 25O$$
$$50O = 400$$
$$O = 8$$
$$Since\ W = O \rightarrow W = 8$$

$\boldsymbol{Eve's\ optimal\ consumption\ bundle\ is\ 8\ glasses\ of\ wine\ and\ 8\ plates\ oumph}$

$$MRT\ is\ the\ slope\ of\ the\ buudget\ constraint, and\ MRS\ is\ the\ slope\ of\ the\ indifference\ curve$$
$$Therefore\ she\ will\ maximize\ her\ utility\ where\ MRS = MRT, i.e,$$
$$where\ the\ indifference\ curve\ is\ tangent\ to\ her\ budget\ constraint.$$

e)

$$\underline{Town\ A}:$$
$$Her\ utility\ function\ is\ the\ same\ as\ before \rightarrow MRS\ is\ till\ the\ same$$
$$The\ prices\ are\ also\ the\ same \rightarrow MRT\ is\ still\ the\ same$$

$$Therefore, W = O\ still\ holds$$

$$She\ now\ has\ a\ new\ budget\ constraint:$$
$$600 = 25W + 25O$$

$$Plot\ W\ into\ the\ new\ budget\ constraint\ and\ solve\ for\ O:$$
$$600 = 25O + 25O$$
$$50O = 600$$
$$O = 12$$
$$W = 12$$

$$Her\ utility\ in\ town\ A\ is\ found\ by\ plotting\ W\ and\ O\ into\ her\ utility\ function:$$
$$Utility = 2 * 12 * 12$$
$$Utility = 288$$

$$\underline{Town\ B}:$$
$$She\ now\ has\ \ a\ new\ MRT, as\ the\ prices\ has\ changed:$$
$$MRT = -\frac{20}{13}$$

$$Set\ MRS = MRT$$

$$-\frac{W}{O}=-\frac{20}{13}$$
$$W=\frac{20}{13}O$$

$$Here\ she\ also\ has\ a\ new\ budget\ constraint:$$
$$400=13W+20O$$

$$Plot\ W\ into\ the\ new\ budget\ constraint\ and\ solve\ for\ O:$$
$$400=13\left(\frac{20}{13}O\right)+20O$$
$$400=20O+20O$$
$$400=40O$$
$$O=10$$
$$W=\frac{20}{13}*10=15{,}38$$

$$Her\ utility\ in\ town\ B\ is\ found\ by\ plotting\ W\ and\ O\ into\ her\ utility\ function:$$
$$Utility=2*10*15{,}38$$
$$Utility\approx 307{,}6$$

$$It\ now\ becomes\ clear\ that\ her\ utility\ is\ higher\ in\ town\ B$$

$$\boldsymbol{She\ will\ move\ to\ town\ B\ if\ utility\ decides\ her\ moving-decision}$$
$$\boldsymbol{In\ town\ A\ she\ will\ consume\ 12\ glasses\ of\ wine\ and\ 12\ plates\ of\ oumph}$$
$$\boldsymbol{In\ town\ B\ she\ will\ consume\ 15{,}38\ glasses\ of\ wine\ and\ 10\ plates\ of\ oumph}$$

3. Duopolies and Game theory

a)

$$This\ situation\ can\ be\ looked\ at\ as\ a\ Cournout\ duopoly:$$
$$Demand: P=150-Q$$
$$MC\ for\ both\ firms=15$$

$$The\ demand\ in\ the\ market\ can\ be\ described\ as:$$
$$P=150-qb-qt$$

$$In\ order\ to\ find\ their\ outputs\ we\ need\ their\ best\ response\ function:$$
$$Best\ response\ is\ found\ by, MR=MC$$

$$Bills\ best\ response:$$
$$To\ find\ MRb\ we\ first\ need\ to\ find\ total\ revenue\ and\ differentiate\ it\ with\ respect\ to\ qb:$$
$$TRb=P*qb$$
$$TRb=(150-qb-qt)*qb$$
$$TRb=150qb-qb^2-qt*qb$$
$$MRb=150-2qb-qt$$

$$Set\ equal\ to\ MC:$$
$$150-2qb-qt=15$$
$$-2qb-qt=-135$$

$$2qb = 135 - qt$$
$$qb = 67{,}5 - \frac{qt}{2}$$

$$Since\ they\ share\ the\ same\ market\ demand\ and\ have\ equal\ MC, symmetry\ implies\ that:$$

$$Teds\ best\ response\ function:$$
$$qt = 67{,}5 - \frac{qb}{2}$$

$$Plot\ qt\ into\ qb\ and\ solve\ for\ qb:$$
$$qb = 67{,}5 - \frac{67{,}5 - \frac{qb}{2}}{2}$$
$$qb = 33{,}75 + \frac{qb}{4}$$
$$\frac{3}{4}qb = 33{,}75$$
$$qb = 45 \rightarrow qt = 45$$
$$Q = 90$$

$$P = 150 - 90 = 60$$

$$\boldsymbol{They\ will\ both\ produce\ 45\ burgers\ each}$$
$$\boldsymbol{The\ price\ will\ be\ 90}$$

b)

$$If\ bill\ was\ the\ leader\ he\ would\ assume\ that\ Ted\ would\ follow\ his\ best\ response:$$
$$qt = 67{,}5 - \frac{qb}{2}$$
$$Plot\ qt\ into\ the\ demand:$$
$$P = 150 - qb - \left(67{,}5 - \frac{qb}{2}\right)$$
$$P = 82{,}5 - \frac{qb}{2}, this\ is\ the\ demand\ Bill\ faces, he\ will\ now\ set\ MRb = MC:$$
$$MRb = 82{,}5 - qb$$
$$82{,}5 - qb = 15$$
$$qb = 67{,}5$$

$$To\ find\ Teds\ quantity\ we\ plot\ qb\ into\ Teds\ best\ response:$$
$$qt = 67{,}5 - \frac{67{,}5}{2}$$
$$qt = 67{,}5 - 33{,}75$$
$$qt = 33{,}75$$

$$P = 150 - 67{,}5 - 33{,}75 = 48{,}75$$

$$\boldsymbol{Bill\ wil\ produce\ 67, 5, and\ Ted\ will\ produce\ 33, 75\ burgers.}$$
$$\boldsymbol{The\ price\ will\ be\ 48, 75}$$

c)

This siutation can be looked at as a Bertrand duopoly:

Bill:

$$Demand: qb = 110 - 2pb + pt$$
$$MCb = 25$$

Ted:

$$Demand: qt = 110 - 2pt + pb$$
$$MCt = 15$$

Bills output:

First we want to find marginal profits. Profits is found by Revenue − TC

$$Revenue = qb * pb$$
$$TC = MC * qb$$

$$pb * (110 - 2pb + pt) - 25 * (110 - 2pb + pt)$$
$$110pb - 2pb^2 + pt * pb - 2750 + 50pb - 25pt$$
$$160pb - 2pb^2 + ptpb - 2750 + 25pt$$

Marginal profits is found by differentiating this equation with respect to pb:

$$MPb = 160 - 4b + pt$$

If we set this equal to 0, we find Bills best response:

$$160 - 4b + pt = 0$$
$$-4pb + pt = -160$$
$$pb = 40 + \frac{pt}{4}$$

Teds output:

We follow the steps above:

$$pt * (110 - 2pt + pb) - 15(110 - 2pt + pb)$$
$$110pt - 2pt^2 + pb * pt - 1650 + 30pt - 15pb$$
$$140pt - 2pt^2 + pbpt - 1650 - 15pb$$

Marginal profits is found by differentiating this equation with respect to pt:

$$MRt = 140 - 4pt + pb$$

If we set this equal to 0, we find Teds best response:

$$140 - 4pt + pb = 0$$
$$-4pt + pb = -140$$
$$pt = 35 + \frac{pb}{4}$$

We can now plot pt into the best response function for Bill:

$$pb = 40 + \frac{35 + \frac{pb}{4}}{4}$$
$$pb = 31{,}25 + 0{,}0625pb$$
$$0{,}9375pb = 31{,}25$$
$$pb = 33{,}33$$

Plot pb into Teds best response:

$$pt = 35 + \frac{33{,}33}{4}$$
$$pt = 43{,}33$$

$$To\ fin\ the\ outputs\ we\ plot\ this\ into\ their\ demands:$$
$$qb = 110 - 2 * (33{,}33) + 43.33 \approx 86.7$$
$$qt = 110 - 2(43{,}33) + 33{,}33 \approx 56{,}7$$

$$\boldsymbol{Ted\ will\ produce\ 56{,}7\ at\ a\ price\ of\ 43{,}33,\ and\ bill\ will\ produce\ 86{,}7\ at\ a\ price\ of\ 33{,}33}$$

d)

$$We\ want\ to\ find\ the\ probability\ of\ each\ of\ them\ choosing\ a\ certain\ strategy:$$

$$Bill\ expected\ payoff - Open\ another\ place:$$
$$\sigma^{TA} * 10 + (1 - \sigma^{TA}) * 70 = 70 - 60\sigma^{TA}$$
$$Peter\ payoff - Not\ opening\ another\ place:$$
$$\sigma^{TA} * 80 + (1 - \sigma^{TA}) * 40 = 40 + 40^{TA}$$
$$Set\ the\ the\ equations\ equal\ to\ each\ other:$$
$$40 + 40\sigma^{TA} \quad = 70 - 60\sigma^{TA}$$
$$100\sigma^{TA} = 30$$
$$\sigma^{TA} = \frac{30}{100}$$

$$Ted\ expected\ payoff\ advertise:$$
$$\sigma^{BOAP} * 90 + (1 - \sigma^{BOAP}) * 20 = 20 + 70\sigma^{BOAP}$$
$$Ted\ expected\ payoff\ not\ advertise\ :$$
$$\sigma^{BOAP} * 30 + (1 - \sigma^{BOAP}) * 60 = 60 - 30\sigma^{BOAP}$$
$$Set\ the\ strategies\ equal\ to\ each\ other:$$
$$60 - 30\sigma^{BOAP} = 20 + 70\sigma^{BOAp}$$
$$100\sigma^{BOAP} = 40$$
$$\sigma^{BOAP} = \frac{40}{100}$$
$$< \sigma^{TA} = \frac{30}{100}, \sigma^{BOAP} = \frac{40}{100} >$$

$$\boldsymbol{Ted\ chooses\ to\ advertise\ with\ a\ probability\ of\ 0{,}3(30\%),\ and}$$
$$\boldsymbol{Bil\ chooses\ to\ open\ another\ place\ with\ the\ probability\ of\ 0{,}4(40\%)}$$