# Part 1:

# Multiple Choice

1. A
2. B
3. C
4. A
5. A
6. C
7. D
8. C
9. A
10. C
11. D
12. A
13. B
14. A
15. B
16. C
17. D
18. B
19. D
20. C

# Part 2:

# Duopolies and Game Theory

I have chosen to answer part 1 about demand side and part 3 about monopoly, duopoly and …

## 1. Demand Side

a)

U = 2OW

To find the marginal utility of wine I differentiate with respect to W:

$$\frac{dU}{dW} = \mathbf{2O}$$

If O double the MU for Wine would also double to be:

$$\frac{dU}{dW}(2(2OW)) = \mathbf{4O}$$

As wine is considered a constant when we differentiate with respect to W, it wouldn't have an impact on the marginal utility if it doubled.

b)

U = 2OW

Y = 400

Price of Oumph = 25

Price of Wine = 25

Step 1. We set up the Lagrangian:

L = 2OW + $\lambda$ ( 400 – 25O – 25W)

Step 2: Find FOC

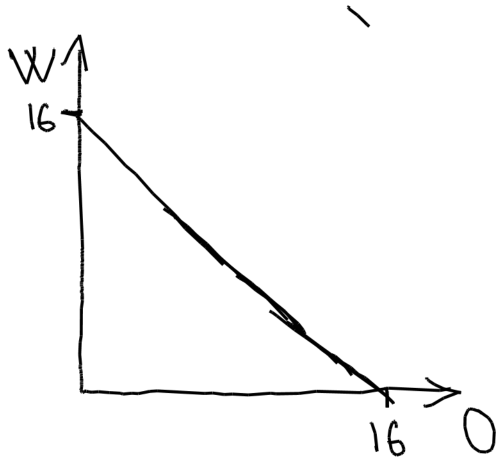
$$\frac{dU}{\quad} = \mathbf{2W - 25\lambda = 0}$$

$dO$

$$\frac{dU}{dW} = \mathbf{2O - 25\lambda = 0}$$

$$\frac{dU}{d\lambda} = \mathbf{400 - 25O - 25W = 0}$$

c)
Budget constraint:




To find MRT I use the formula:

$$-\frac{p_1}{p_2} = \frac{-25}{25} = \mathbf{-1}$$

The slope of the budget constraint is, therefore, -1 which can also be seen from the graph.

d)

I continue my Lagrangian.

Step 3: Isolate $\lambda$ in first to equations

$$\mathbf{2W = 25\lambda}$$

$$\mathbf{2O = 25\lambda}$$

Step 4: We divide the two equations with each other as they both equal $\lambda$

$$\frac{2W}{2O} = \frac{25\lambda}{25\lambda} => \frac{W}{O} = 1$$

Step 5: We isolate W:

W = 1O

Step 6: We substitute our value for W into the budget constraint and isolate O

400 – 25O – 25(1O) = 0

400 – 50O = 0

400 = 50O

**8 = O**

Step 6: We substitute our value for O into our function for W

W = 1(8)

**W = 8**

Eve's optimal consumption will therefore be **8 Oumphs and 8 Wines.**

To find MRS, I use the formula

$$-\frac{MU_O}{MU_W} => \frac{-2W}{2O} = \frac{-16}{16} = \mathbf{-1}$$

As we found out in the earlier assignment, MRT was also equal to -1, which means that.

MRS = MRT in Eve's optimal bundle.

e)

I start of by doing the Lagrangian for Town A:

Step 1. We set up the Lagrangian:

L = 2OW + $\lambda$ ( 600 – 25O – 25W)

Step 2: Find FOC

$$\frac{dU}{dO} = \mathbf{2W - 25\lambda = 0}$$

$$\frac{dU}{dW} = \mathbf{2O - 25\lambda = 0}$$

$$\frac{dU}{d\lambda} = \mathbf{600 - 25O - 25W = 0}$$

Step 3: Isolate $\lambda$ in first to equations

$\mathbf{2W = 25\lambda}$

$\mathbf{2O = 25\lambda}$

Step 4: We divide the two equations with each other as they both equal $\lambda$

$$\frac{2W}{2O} = \frac{\mathbf{25\lambda}}{\mathbf{25\lambda}} => \frac{W}{O} = 1$$

Step 5: We isolate W:

W = 1O

Step 6: We substitute our value for W into the budget constraint and isolate O

600 – 25O – 25(1O) = 0

600 – 50O = 0

600 = 50O

**12 = O**

Step 6: We substitute our value for O into our function for W

W = 1(12)

**W = 12**

Eve's optimal consumption in town A will therefore be **12 Oumphs and 8 Wines**, Which gives her a utility of:

U = 2(12)*12

U = 24*12

**U = 288**

I now do the Lagrangian for Town B

Step 1. We set up the Lagrangian:

L = 2OW + $\lambda$ ( 400 – 20O – 13W) Step

2: Find FOC

$$\frac{\mathrm{dU}}{\mathrm{dO}} = 2\mathrm{W} - 20\lambda = 0$$

$$\frac{\mathrm{dU}}{} = 2\mathrm{O} - 13\lambda = 0$$

dW

dU

$\_\_ = 400 - 20O - 13W = 0$ d$\lambda$

Step 3: Isolate $\lambda$ in first to equations

$2W = 20\lambda$

$2O = 13\lambda$

Step 4: We divide the two equations with each other as they both equal $\lambda$

$$\frac{2W}{2O} = \frac{20\lambda}{13\lambda} => \frac{W}{O} = 1{,}54\ (rounded)$$

Step 5: We isolate W:

W = 1,54O

Step 6: We substitute our value for W into the budget constraint and isolate O

400 – 20O – 13(1,54O) = 0

400 – 20O -20,02 = 0

400 = 40O

**10 = O**

Step 6: We substitute our value for O into our function for W

W = 1,54(10)

**W = 15,4**

Eve's optimal consumption in town B will therefore be **10 Oumphs and 15,4 Wines**, Which gives her a utility of:

U = 2(10)*15,4

U = 20*15,4

**U = 308**

**I can therefore conclude bases on these calculations that Eve will choose to move to town B, as she will receive a higher utility there compared to Town A.**

## 3. Duopolies and Game Theory

a)

When they choose output simultaneously, I will use the Corunot model to find the price:

We are given THE following information:

P = 150-Q MC

= 15

$q_1$ = Bill $q_2$

= Ted

Step 1: Find the inverses residual demand curve:

P = 150 - ($q_1$ + $q_2$)

Step 2:

Find Total revenue for firm 1 and differentiate to find MR

TR = 150 - ($q_1$ + $q_2$) * $q_1$

TR = 150$q_1$ – $q_1^2$ –$q_1$*$q_2$

MR = 150 - 2$q_1$ – $q_2$

Step 3: Set MR = MC and solve for $q_1$ (Best response function for Bill)

MR = MC => 150 - 2$q_1$ – $q_2$ = 15 => 135 – $q_2$ = 2$q_1$ $q_1$ = 67,5 - $\frac{1}{2}q_2$

Step 4: Do the exact same thing for firm 2 (Find Ted's best response)

As they have the same demand and same MC, we now that Ted's best response function will be:

$q_2$ = 67,5 - $\frac{1}{2}q_1$

Step 5: Plug $q_2$ into Bill's best response function and solve for $q_1$

$q_1$ = 67,5 $\frac{1}{2}$ - $(67,5 - \frac{1}{2}$
$q_1)$

= 67,5 - $33,75 - \frac{1}{4} q_1$

$\frac{3}{4} q_1 = 33,75$

**$q_1$ = 45**

We therefore also know that:

**$q_2$ = 45** (Because they have the same functions)

Step 6: Plug $q_1$ and $q_2$ into P

P = 150 – (45+45)

**P = 60**

**This means they will both produce 45 burgers to a price of 60DKK**

b)

Now that there is a leader and a follower, I will use the Stackelberg model to find the equilibrium:

$P = 150-(q_1+q_2)$ MC

= 15

$q_1$ = Bill (Leader) $q_2$

= Ted (follower)

Step 1: We find the followers best response function ($q_2$)

$TR_2 = 150-(q_1+q_2) *q_2$

$TR_2 = 150q_2 – q_2^2 – q_1*q_2$

$MR_2 = 150 -2q_2 – q_1$

$MR_2 = MC => 150 -2q_2 – q_1 = 15 => 135 – q_1 = 2q_2$

$\mathbf{q_2 = 67{,}5 - \frac{1}{2} q_1}$ **(Followers best response function)**

Step 2: Insert $q_2$ into leader's residual demand function to find leaders output

$P = 150 - q_1 + 67{,}5 - \frac{1}{2}q_1)$

$P = 217{,}5 - 1\frac{1}{2}q_1$

Step 3: Find leaders MR and MC and solve them for $q_1$ to find output for leader

$TR_1= (217{,}5 – 1{,}5q_1) * q_1$

$MR_1 = 217{,}5 – 3q_1$ (When we have a linear function for TR, we can just multiply the slope by 2 to find MR) $MR_1 = MC$

$= > 217{,}5 – 3q_1 = 15 => 202{,}5 = 3q_1$ $\mathbf{q_1 = 67.5}$

Step 4: Insert $q_1$ value into follower's best response function to find followers output

$q_2 = 67{,}5 - \frac{1}{2}(67{,}5)$ $\mathbf{q_2 = 33{,}75}$

**This means that Bill (leader) will sell 67,5 Burgers while Ted (follower) will sell only 33,75. The total market quantity is therefore 105,25 (67,5 + 37,75).**

They will both sell at a price of:

$P = 150 – (67{,}5 + 37{,}75)$

**P = 44.75**

**The market equilibrium is therefore (105,25;44,75) in the Stackelberg duopoly.**

c)

As they now compete on choosing their prices, we will use the Bertrand model to find output and price.

We are given the following information:

$q_1 = 110 – 2p_1 – p_2$ $MC_1 = 25$ $q_2 = 110 –$

$2p_2 – p_1$ $MC_2 = 15$

$p_1$ = Bill

$p_2$ = Ted

Step 1: Set up the profit function for firm 1 as a function of $p_1$

$Profit_1 = p_1 * q_1 - TC_1$

$Profit_1 = (110 – 2p_1 – p_2) * p_1 – 25(110 – 2p_1 – p_2)$

$Profit_1 = 110p_1 – 2p_1^2 – p_1*p_2 – 2750 – 50p_1 – 25p_2$

$MP_1 = 110 – 4p_1 – p_2 – 50$

Step 2: Set marginal profits equal to 0 and isolate $p_1$ to find firm 1 best response function

$MP_1 = o$

$110 – 4p_1 – p_2 – 50 = 0$

$160 – p_2 = 4p_1$

$\mathbf{p_1 = 40 - \frac{1}{4} p_2}$ **(Bills best response function)**

Step 3: We do the same thing for firm 2

$Profit_2 = p_1 * q_1 - TC_1$

$Profit_2 = (110 – 2p_2 – p_1) * p_2 – 15(110 – 2p_2 – p_1)$

$Profit_2 = 110p_2 – 2p_2^2 – p_1*p_2 – 1650 – 30p_2 – 15p_1$

$MP_2 = 110 – 4p_2 – p_1 – 30$

$MP_2 = 0$

$110 - 4p_2 - p_1 - 30 = 0$

$140 - p_1 = 4p_2$

**$p_2 = 35 - \frac{1}{4}p_1$ (Ted's best response function)**

Step 3: We insert $p_2$ into $p_1$ and find the value for $p_1$

$40 - \frac{1}{4}(35 - \frac{1}{4}p_1)$ $p_1 = 40 - 8,75 + \frac{1}{8}p_1$ $\frac{7}{8}p_1 = 31,25$ **$p_1$ = 35,71 (Bill's price)**

Step 4: We insert our $p_1$ value into our best response function for $p_2$

$p_2 = 35 - \frac{1}{4}(35,71)$ $p_2 = 35 - 8,93$ **$p_2$ = 26,07 (Ted's price)**

Step 5: Plug $p_1$ and $p_2$ into our demand functions

$q_1 = 110 - 2(35,71) - 26,07 =$ **12,51** $q_2 = 110 - 2(26,07) - 35,71 =$ **22,15**

**This means that Bill will sell 12,51 burger at a price of 35,71DKK.**

**Ted will sell 22,15 burgers at a price of 26,07DKK.**

d)

As we can see from the model below, there is not a true Nash equilibrium but a mixed Nash equilibrium:

| | | Bill: Open | Bill: not open |
|---|---|---|---|
| Ted | Ad | 90, 10 | 20, 80 |
| | No ad | 30, 70 | 60, 40 |

I let $\theta$t be the probability that Ted advertises and I let (1-$\theta$t) be the probability that he doesn't advertise.

$\theta$b is the probability Bill opens another shop and (1-$\theta$t) is the probability he doesn't open another shop.

I want Bills expected payoff to be the same whether he chooses to open or not to open.

Expected payoff if opening = $\theta_t * 10 + (1 - \theta_t) * 70 => 10\theta_t + 70 - 70\theta_t$ Expected

payoff if not opening = $\theta_t * 80 + (1 - \theta_t) * 40 => 80\theta_t + 40 - 40\theta_t$

I set these to equal and isolate $\theta_t$:

$10\theta_t + 70 - 70\theta_t = 80\theta_t + 40 - 40\theta_t$

$30 = 100\theta_t => \boldsymbol{\theta_t} = \frac{\mathbf{3}}{\mathbf{10}}$ **(Probability that Ted advertises)**

$(1 - \frac{3}{10}) = \frac{\mathbf{7}}{\mathbf{10}}$ **(Probability that Ted doesn't advertise)**

I do the same thing for Ted:

Expected payoff if advertising: $\theta_B * 90 + (1 - \theta_B) * 20 => 90\theta_B + 20 - 20\theta_B$

Expected payoff if not advertising: $\theta_t * 30 + (1 - \theta_t) * 600 => 30\theta_t + 60 - 60\theta_t$
I set these to equal and isolate $\theta_B$:

$90\theta_B + 20 - 20\theta_B = 30\theta_t + 60 - 60\theta_t$

$40 = 100\theta_t => \boldsymbol{\theta_B} = \frac{\mathbf{4}}{\mathbf{10}}$ **(Probability that Bill opens another shop)**

$(1 - \frac{4}{10}) = \frac{\mathbf{6}}{\mathbf{10}}$ **(Probability that Bill doesn't open a shop)**